

1. If $f(x) = 3x - 1$ and $g(x) = x^2$, find $g(f(f(1)))$.
A. 1 B. 4 C. 5 D. 16 E. 25
2. The teachers at Oak Tech have cars with average mileage 39000 miles. George buys a brand-new car, keeping his old car, and the average mileage drops to 36400. How many cars do the teachers now own?
A. 12 B. 13 C. 14 D. 15 E. 16
3. The sequence $\log x, \log x^2, \log x^3, \log x^4, \dots$ is best described as which of the following?
A. geometric with common ratio $\log x$ B. geometric with common ratio x
C. arithmetic with common difference $\log x$ D. arithmetic with common difference x
E. neither geometric nor arithmetic
4. A set of seven different positive integers has mean and median both equal to 20. What is the largest possible value this set can contain?
A. 65 B. 67 C. 71 D. 73 E. 77
5. If $AM/AT = .YC$, where each letter represents a different digit, AM/AT is in simplest terms, and $A \neq 0$, then $AT =$
A. 15 B. 16 C. 25 D. 28 E. 75
6. A sheet of stamps is five stamps high and four stamps wide. Each stamp is 2 inches wide and 1 inch high. If a connected group of five stamps is torn from the sheet, let P be the largest possible perimeter and p the smallest possible perimeter for the torn-out group. Find P/p .
A. $\frac{4}{3}$ B. $\frac{11}{8}$ C. $\frac{11}{7}$ D. $\frac{10}{7}$ E. $\frac{3}{2}$
7. If $\ln s = 0.6$ and $\ln t = 0.9$, find $\log_{st} e^{5.4}$.
A. 3.6 B. 5 C. 5.4 D. 10 E. 10.8
8. A function f is symmetric to the origin and periodic with period 8. If $f(2) = 3$, what is the value of $f(4) + f(6)$?
A. -6 B. -3 C. 0 D. 3 E. 6
9. For how many integer values of k do the graphs of $x + y = k$ and $xy = k$ NOT intersect?
A. 0 B. 1 C. 2 D. 3 E. more than 3
10. A point is chosen at random from the interior of a square of side 16. Find the probability that the point is at least $\sqrt{2}$ units from both diagonals.
A. $\frac{9}{16}$ B. $\frac{5}{8}$ C. $\frac{3}{5}$ D. $\frac{3}{4}$ E. $\frac{4}{9}$
11. The graph of the function $f(x) = x + \sin kx$ ($|k| \leq 1$) intersects the graph of the function $f^{-1}(x)$ at $(4,a)$, $(12,b)$, and $(-8,c)$. Find the value of $a + b + c$.
A. 4 B. 8 C. 10 D. 12 E. 16

12. If $\cos(\arctan(x)) = x$ (x in radians), then x^2 can be expressed in the form $\frac{a+\sqrt{b}}{2}$. Find $a + b$.
 A. 4 B. 5 C. 6 D. 7 E. 8
13. A jug holds 10 gal of antifreeze. I fill an empty bottle from the jug and replace the amount I poured out with water, mixing well. I refill the emptied bottle again from the jug, refilling the jug with water and mixing well, and then repeat this process once more. The bottle is now half water. To the nearest tenth of a gallon, what is the volume of the bottle?
 A. 2.3 B. 2.5 C. 2.7 D. 2.9 E. 3.1
14. How many different 3-letter strings can be formed from the letters of MATHEMATICS (no letter can be used in a given string more times than it appears in the word)?
 A. 336 B. 399 C. 660 D. 675 E. 990
15. A farmer plants A acres of wheat one year. Each year thereafter, he harvests (removes) $1/4$ of the planted acreage and then plants 1500 more acres. The number of acres of wheat planted approaches what number?
 A. 3000 B. 4000 C. 5000 D. 6000 E. it depends on A
16. Right $\triangle ABC$ (right angle at B) has legs of length 68 and 285. If the medians from vertex A and vertex C intersect at D , find the area of $\triangle ADC$ to the nearest ten square units.
 A. 3220 B. 3230 C. 3240 D. 3250 E. 3260
17. If $f(x) = \frac{x^2 - 3x - 4}{x + 1}$, the inverse of $f(x)$ can be written as $f^{-1}(x) = \frac{x^2 + ax + b}{x + c}$. Find $a + b + c$.
 A. -14 B. -2 C. 4 D. 10 E. 34
18. Choose k so that the system $\begin{cases} x + y + kz = 1 \\ x + ky + z = 2 \\ kx + y + z = -3 \end{cases}$ is dependent. For which pair (x, y) below does there exist a z such that (x, y, z) will satisfy the resulting dependent system?
 A. $\left(\frac{7}{3}, 0\right)$ B. $\left(3, \frac{2}{3}\right)$ C. $\left(\frac{8}{3}, 1\right)$ D. $\left(\frac{4}{3}, -1\right)$ E. $\left(\frac{1}{3}, -2\right)$
19. A pentagon is circumscribed about a circle of diameter 6 (that is, each side of the pentagon is tangent to the circle). If the pentagon has area 42 cm^2 , find its perimeter in centimeters.
 A. 14 B. 21 C. 24 D. 28 E. 35
20. The sum of the solutions of $\arctan \frac{1}{x} + \arctan \frac{1}{x+2} = \arctan \frac{4}{x+4}$ is
 A. negative B. even C. 1 D. greater than 5 E. prime

PLEASE PRINT!

AMATYC STUDENT MATHEMATICS LEAGUE

COLLEGE: KEY – SPRING 2007 (CORRECTED) STATE: _____

LAST NAME:

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GENDER: Male Female

FIRST NAME:

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EMAIL ADDRESS: _____ INSTRUCTOR _____

DO YOU HAVE A TWO-YEAR COLLEGE OR HIGHER DEGREE FROM ANY SCHOOL IN THE WORLD? YES NO

	Student's Responses	Local Corrector
1	E	
2	D	
3	C	
4	C	
5	C	
6	D	
7	A	
8	B	
9	D	
10	A	
11	B	
12	A	
13	D	
14	B	
15	D	
16	B	
17	E	
18	C	
19	D	
20	E	

ROUND: 1 2

correct = _____

incorrect = _____

blank = _____

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 = # correct × 2

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 = # wrong × $\frac{1}{2}$

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 = score